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Enrichment of the contact geometry within the Finite Element Method

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Résumé — A new approach for sub-mesh enrichment of the contact geometry has been proposed in the framework of the Finite Element Method and the Node-to-Segment contact discretization. The method is very general and multipurpose : it allows, for example, to extend the limits of the contact modeling of thin-walled structures or to account for a change of the contact geometry due to loading, for example, to simulate a shallow wear.

Mots clés — Contact, Node-to-Segment contact discretization, Finite Element Method, multiscale approach, enrichment of shape functions, smoothing techniques.

1 Introduction

Contact occurs at the interface between bodies. In the Finite Element Method this interface is discretized by nodes and segments. In the classical master-slave approach and the Node-to-Segment (NTS) discretization, this interface is represented by so-called "contact elements", which consist of one slave node and one or several master segments. In general, the contact geometry of such an element is fully described by the interpolation function of the master segment or by a smoothing function aimed to represent a C^1 geometry over several master segments. However the contact geometry can be enriched independently of the finite element mesh and the related interpolation functions.

In the second half of the 90s several approaches based on enriching of the element interpolation functions have been proposed for different problems [2], [1]. The entire class of enriched or extended finite element methods got the name XFEM (extended finite element method) [8] or GFEM (generalized finite element method) or PUM (partition of unity method) [7]. This method is used for modeling of dislocations, solidification, two-fluid flows, cracks and cohesive cracks.

We propose to enrich the geometry of the master surface in a rather similar manner (Fig. 1), as the solution field is enriched within the XFEM, so that the master surface $\underline{\rho}$ within the contact element is described by the following equation :

$$\underline{\rho}_e = \underline{\rho} + h_e \underline{n} \quad \Leftrightarrow \quad \underline{\rho}_e(\underset{\sim}{\xi}) = \underline{\rho}(\underset{\sim}{\xi}) + h_e(\underset{\sim}{\xi}, \underset{\sim}{\Theta})\underline{n}(\underset{\sim}{\xi}), \tag{1}$$

where $\underline{\rho}$ and $\underline{\rho}_e$ are vectors describing the original and enriched master surfaces respectively, $\underline{n}$ is a unit normal vector to the original master surface and $h_e(\underset{\sim}{\xi}, \underset{\sim}{\Theta})$ is an enriching function which depends on the convective coordinate of the master segment(s) $\underset{\sim}{\xi}$ and, in general, may depend on the local strain-stress state and its history. So this approach can be used, for example, to account for geometry change due to wear, deformation of asperities, dislocation escape, relaxation, etc. This dependence is taken into account by means of an array of variables $\underset{\sim}{\theta}$, which can also include time. Remark that the single and double line underlined quantities are vectors and second-order tensors respectively, quantities underlined by a wave represent an ordered array of scalar quantities (e.g., two convective coordinates $\underset{\sim}{\xi} = [\xi_1, \xi_2]^T$, array of parameters $\underset{\sim}{\theta}$, etc.), quantities underlined by a double wave represent matrices of scalars (e.g., a surface metric matrix), the quantities underlined by a line and a wave designate ordered arrays of vectors (e.g., a set of basis vectors), see [11] and [12].

There are several motivations to enrich the contact geometry : 1) a multiscale modeling of contact with thin-walled structures with a complex geometry of the surface as well as modeling of contact with a soft bodies with hard coatings ; 2) implicit modeling of anisotropic friction ; 3) account of a complex change of the surface geometry due to loading, etc. The main advantage of the method is an enrichment of the geometry on the sub-mesh scale without increasing the computational costs. The main drawback is a challenging analytical formulation of the geometrical quantities entering in formulae for the residual vector and the tangent matrix of contact elements.

First of all, a general weak form for contact problems will be presented. Next some remarks on the derivation of main equations will be given. In conclusion, some perspectives and applications will be proposed.

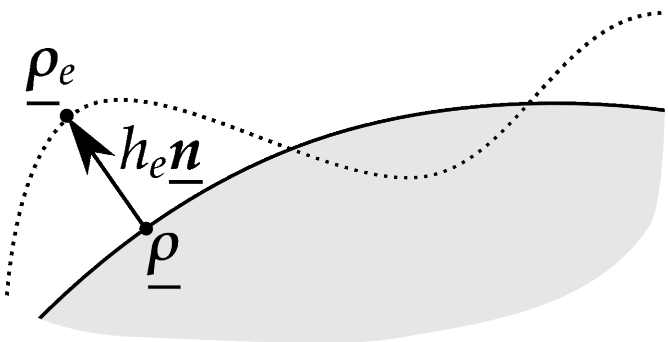

FIG. 1 – Enriched geometry $\underline{\rho}_e$ of the master surface $\underline{\rho}$.

2 Weak formulation of the contact problem

The Finite Element Method is based on the integral (weak) form of the equilibrium equation. A rigorous construction of a weak form for the contact problems results in a variational inequality [3]. However this formulation is hard to apply for the case of nonlinear material, large deformation and large sliding contact. That is why for the general case it is advantageous to use the approach of variational equalities [9], assuming known in advance the active contact zone Γ_c :

$$\begin{gathered}\int\limits_{\Omega} \underline{\underline{\sigma}} \cdot\cdot\, \delta \underline{\underline{\varepsilon}}\, d\Omega + \int\limits_{\Gamma_c} \delta W_c\, d\Gamma_c = \int\limits_{\Gamma_f} \underline{\sigma_0} \cdot \delta \underline{u}\, d\Gamma + \int\limits_{\Omega} \underline{f_v} \cdot \delta \underline{u}\, d\Omega, \\ \mathbb{V} = \left\{ \delta \underline{u} \in \mathbb{H}^1(\Omega) \;\middle|\; \delta \underline{u} = 0 \text{ on } \Gamma_u \right\}\end{gathered}, \tag{2}$$

where $\underline{\underline{\sigma}}$ is the Cauchy stress tensor, $\underline{\underline{\varepsilon}}$ is the Green-Lagrangian strain tensor, Ω is the interior of contacting bodies in the current configuration, $\underline{\sigma_0}$ is a prescribed traction on the surface Γ_f and $\underline{f_v}$ is a vector field of volume force density. The displacement vector field $\underline{u}$ belongs to the Hilbert space $\mathbb{H}^1(\Omega)$ and fulfills the Dirichlet boundary conditions $\underline{u} = \underline{u}_0$ on the boundary Γ_u. The form of the term δW_c, which has to be integrated over the contact area Γ_c of any of contacting surfaces, depends upon the choice of the resolution method (penalty, Lagrange multiplier, augmented Lagrangian methods and others) and in general can be written as a linear function of four variations :

$$\delta W_c = A \delta g_n + \underset{\sim}{B}^T \delta \underset{\sim}{\xi} + C \delta \lambda_n + \underset{\sim}{D}^T \delta \underset{\sim}{\lambda}_t, \tag{3}$$

where g_n is the local normal gap between contacting surfaces ($g_n > 0$ denotes that there is no contact, $g_n = 0$ means that bodies are in contact and $g_n < 0$ designates a local penetration), $\underset{\sim}{\xi}$ is the convective parameter describing the projection point of the slave node on the master segment, λ_n is the Lagrange multiplier representing the contact pressure and $\underset{\sim}{\lambda}_t$ is a set of Lagrange multipliers representing the components of the tangential contact stress vector in the local basis $\dfrac{\partial \underline{\rho}}{\partial \underset{\sim}{\xi}}$; A, C are scalar functions of geometrical quantities and the local stress state and $\underset{\sim}{B}$, $\underset{\sim}{D}$ are components of vector functions in the local basis. The linearization of the contact term (3) from Eq. (2) needed in an implicit resolution scheme, requires the variation of the virtual work term δW_c :

$$\Delta \delta W_c = \frac{\partial \delta W_c}{\partial g_n} \Delta g_n + A \Delta \delta g_n + \frac{\partial \delta W_c}{\partial \underset{\sim}{\xi}}^T \Delta \underset{\sim}{\xi} + \underset{\sim}{B}^T \Delta \delta \underset{\sim}{\xi} + \frac{\partial \delta W_c}{\partial \lambda_n} \Delta \lambda_n + \frac{\partial \delta W_c}{\partial \underset{\sim}{\lambda}_t}^T \Delta \underset{\sim}{\lambda}_t \tag{4}$$

The closed form expressions for variations δg_n, $\delta \underset{\sim}{\xi}$, $\Delta \delta g_n$, $\Delta \delta \underset{\sim}{\xi}$ for the continuous geometry independent on discretization were first obtained in [6]. The terms $\frac{\partial \delta W_c}{\partial \bullet}$ depend on the resolution method and remain unchangeable in case of enrichment of the contact geometry. However the variations of the geometrical quantities have to be recalculated.

3 Main equations for the enriched contact element

To derive a consistent framework one has to impose two conditions on the enriching function $h_e(\underset{\sim}{\xi}, \underset{\sim}{\Theta})$:

– $h_e(\underset{\sim}{\xi}, \underset{\sim}{\Theta}) \in C^2(\underset{\sim}{\xi}) \cap C^1(\underset{\sim}{\theta})$, i.e. $h_e(\underset{\sim}{\xi}, \underset{\sim}{\Theta})$ is a C^2-smooth function by convective coordinate $\underset{\sim}{\xi}$ and C^1-smooth by $\underset{\sim}{\theta}$;

– in order to avoid self-intersection of the enriched surface, we require that the value of enriching function remains smaller than the minimal local curvature radius of the surface $|h_e(\underset{\sim}{\xi})| < \min\limits_i \left\{1/\kappa_i(\underset{\sim}{\xi})\right\}$.

If the master geometry is enriched locally (within each master segments), in order to avoid gaps in the surface these conditions have to be complemented by a third one : the value of the enriching function has to be zero at the boundary of the contact element $h_e(\partial\Omega^e) = 0$. The latter condition is very restrictive and renders a mesh dependent method. This difficulty can be overcome if instead of local enriching, one enriches the smoothing surface constructed over several master segments, see Fig. 2.

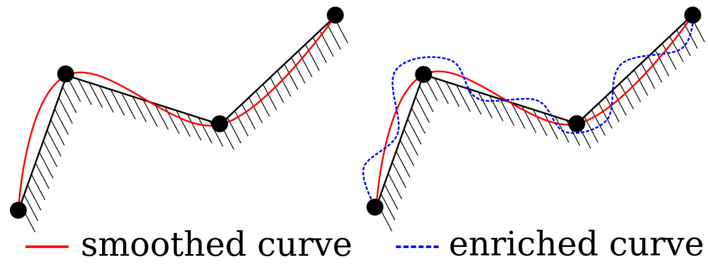

Fig. 2 – Enriched geometry of the smoothed master surface.

If the enriching function depends only on the surface parameter (i.e. it does not change in time and does not depend on the deformation state) or if it changes very slowly, then all the variations of the kinematic quantities remain the same as in [6] if one replaces all quantities related to the master surface by enriched ones $\bullet_e$:

$$\underline{\rho}_e = \underline{\rho} + h_e\underline{n}, \quad \underline{n}_e = \frac{\dfrac{\partial \underline{\rho}_e}{\partial \xi_1} \times \dfrac{\partial \underline{\rho}_e}{\partial \xi_2}}{\left\| \dfrac{\partial \underline{\rho}_e}{\partial \xi_1} \times \dfrac{\partial \underline{\rho}_e}{\partial \xi_2} \right\|}, \quad \bar{\delta}\underline{\rho}_e = \delta\underline{\rho}_e + \frac{\partial \underline{\rho}_e}{\partial \underset{\sim}{\xi}}^T \delta\underset{\sim}{\xi}$$

$$\delta\underline{\rho}_e = \delta\underline{\rho} + h_e\delta\underline{n}, \quad \frac{\partial \underline{\rho}_e}{\partial \underset{\sim}{\xi}} = \frac{\partial \underline{\rho}}{\partial \underset{\sim}{\xi}} + \underline{n}\frac{\partial h_e}{\partial \underset{\sim}{\xi}} + h_e\frac{\partial \underline{n}}{\partial \underset{\sim}{\xi}}, \quad \underset{\approx}{A}_e = \frac{\partial \underline{\rho}_e}{\partial \underset{\sim}{\xi}} \cdot \frac{\partial \underline{\rho}_e}{\partial \underset{\sim}{\xi}}^T, \quad \underset{\approx}{H}_e = \underline{n}_e \cdot \frac{\partial^2 \underline{\rho}_e}{\partial \underset{\sim}{\xi}^2}$$

$$\frac{\partial^2 \underline{\rho}_e}{\partial \underset{\sim}{\xi}^2} = \frac{\partial^2 \underline{\rho}}{\partial \underset{\sim}{\xi}^2} + \frac{\partial \underline{n}}{\partial \underset{\sim}{\xi}}\frac{\partial h_e}{\partial \underset{\sim}{\xi}}^T + \frac{\partial h_e}{\partial \underset{\sim}{\xi}}\frac{\partial \underline{n}}{\partial \underset{\sim}{\xi}}^T + h_e\frac{\partial^2 \underline{n}}{\partial \underset{\sim}{\xi}^2} + \underline{n}\frac{\partial^2 h_e}{\partial \underset{\sim}{\xi}^2}$$

The explicit forms of these equations can be found in [11]. However to incorporate these modified quantities in a Finite Element framework one has to derive the equations for the basic variations of the enriched master vector and its derivatives

$$\delta \underline{\rho_e}, \quad \delta \frac{\partial \underline{\rho_e}}{\partial \underset{\sim}{\xi}}, \quad \delta \frac{\partial^2 \underline{\rho_e}}{\partial \underset{\sim}{\xi}^2}.$$

If the first expression is easily derivable, however to obtain the two latter, a significant efforts have to be undertaken [11]. However if one confies himself to the case of infinitely small gaps $g_n = 0$, as, for example, in [4], the formulation becomes much simpler, since in this case there is no need to derive $\delta \frac{\partial^2 \underline{\rho_e}}{\partial \underset{\sim}{\xi}^2}$. It is worth noting that the quantities $\delta \frac{\partial \underline{\rho_e}}{\partial \underset{\sim}{\xi}}$ and $\delta \frac{\partial^2 \underline{\rho_e}}{\partial \underset{\sim}{\xi}^2}$ enters in equations for the variations of the geometrical quantities as dot products with the normal vector to the enriched surface $\underline{n}_e$ and its local basis $\frac{\partial \underline{\rho_e}}{\partial \underset{\sim}{\xi}}$, which are in turn can be expressed by the original normal vector $\underline{n}$ and the local basis $\frac{\partial \underline{\rho}}{\partial \underset{\sim}{\xi}}$:

$$\delta \frac{\partial \underline{\rho_e}}{\partial \underset{\sim}{\xi}} \quad \rightarrow \quad \underline{n} \cdot \delta \frac{\partial \underline{\rho_e}}{\partial \underset{\sim}{\xi}}, \ \frac{\partial \underline{\rho}}{\partial \underset{\sim}{\xi}} \cdot \delta \frac{\partial \underline{\rho_e}}{\partial \underset{\sim}{\xi}}^T; \quad \delta \frac{\partial^2 \underline{\rho_e}}{\partial \underset{\sim}{\xi}^2} \quad \rightarrow \quad \underline{n} \cdot \delta \frac{\partial^2 \underline{\rho_e}}{\partial \underset{\sim}{\xi}^2}, \ \frac{\partial \underline{\rho}}{\partial \underset{\sim}{\xi}} \cdot \delta \frac{\partial^2 \underline{\rho_e}}{\partial \underset{\sim}{\xi}^2}^T,$$

the latter quantities are significantly easier to derive than the original ones. In general case, in order to avoid these complicated derivations and their programming, one can incorporate in the employed finite element software, an automatic procedure which derives the needed quantities and constructs the residual vector and the tangent matrix for contact elements. This approach has been proposed in [10] and [5], where the *AceGen* software, based on the package of symbolic mathematical calculations *Mathematica*, has been used to construct the needed quantities for frictional contact in case of smoothed master surface.

It is worth mentioning that the detection procedure has to be also changed. It is often based on the detection of the normal projection of a slave node on the master surface. Obviously in case of enriched geometry the projection point has to be determined on the enriched surface (Fig 3).

As already mentioned, the enriching function h_e has to be rather smooth (C^2) and its value $|h_e|$ must remain smaller than the minimal local curvature radius. Moreover if one uses the normal projection, it is necessary to keep in mind that Newton's method, often used for this purpose, allows to determine only one projection point (closest to the starting point), that is why the function h_e has to be chosen properly in order to avoid multiple solutions of $\min F(\underline{r}_s, \underset{\sim}{\xi})$ (Fig. 4 a), where

$$F(rs, \underset{\sim}{\xi}) = \frac{1}{2}(\underline{r}_s - \underline{\rho}(\underset{\sim}{\xi})^2$$

is a distance function. Contrary to the normal projection, the shadow projection proposed in [11] is unique if there is no "shadows" from the master surface on its own, so one has to pay attention to avoid shadows due to an enrichment of the master surface (Fig. 4 b).

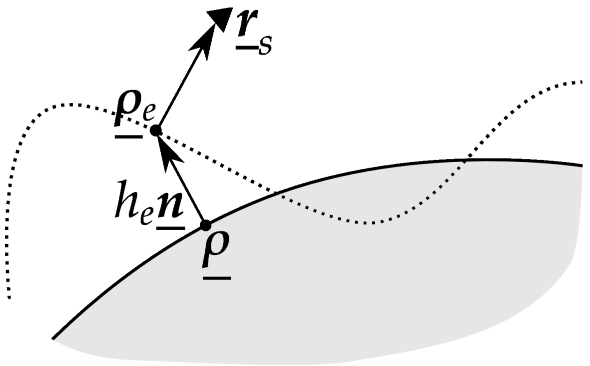

FIG. 3 – The procedure of the normal projection in case of an enriched master geometry ($\underline{r}_s$ is a slave node position).

As already mentioned, in order to preserve the continuity of the master surface in case of local enrichment, enriching functions have to be zero at edges of each master segment $h_e(\partial\Omega^e) = 0$ (Fig. 4 c). It has to be also mentioned that there is a possibility of intersection of enriched geometries of adjacent master segments, which has to be avoided as well (Fig. 4 d).

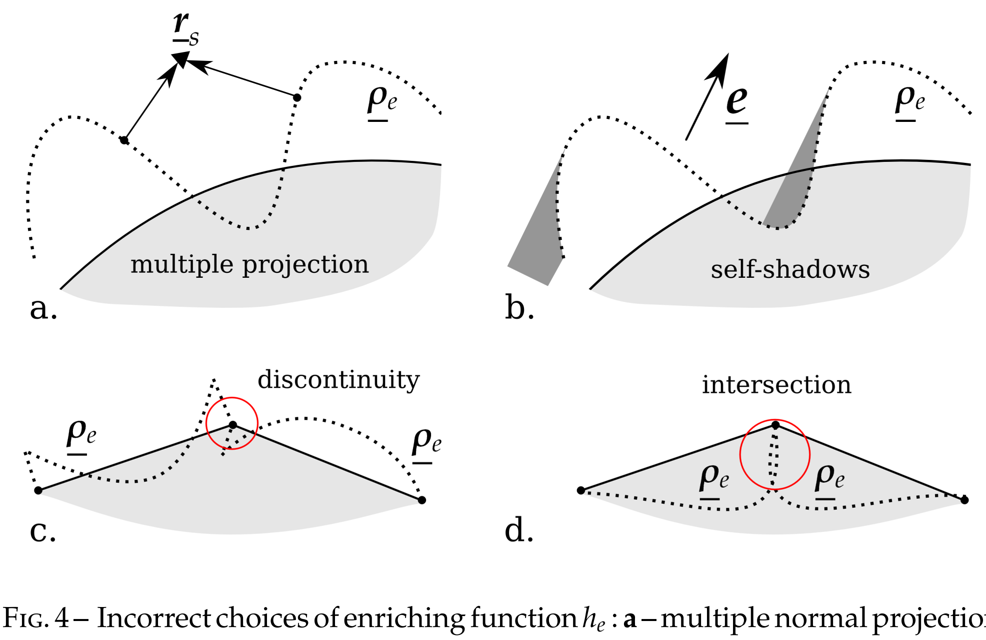

FIG. 4 – Incorrect choices of enriching function h_e : **a** – multiple normal projection within one segment ; **b** – presence of self-shadows for shadow projection ; **c** – discontinuity of enriched master surface ; **d** – self intersection of enriched master surface.

4 Perspectives and applications

The enrichment of the contact geometry by an arbitrary function permits :

1. to take into account a complicated geometry within one contact element ;
2. to account for a change of the local geometry due to loading conditions.

As mentioned, if the enrichment is chosen to be localized within NTS contact elements, the choice of the enrichment function is limited : its value must be zero at the edges of the master segments. It implies a strong connection between the finite element mesh and the enrichment. A possible application of this approach is the modeling of periodic structures using a regular mesh, Fig. 5. Enrichment of thin-walled or beam structures by a constant enriching function seems to be meaningfull, since the predominant deformation of such structures does not affect the geometry of the surface (Fig. 5,a-b). A possible applications is a modeling of contact with grid structures, micro contact with fiber, etc. The proposed approach is also valid for the case, when the deformation of the surface geometry is small in comparison to the deformation of the bulk material (Fig. 5,c-d), for instance, hard coating on a soft substrate. An anisotropic friction can be simulated implicitly by a special enrichment of the master surface (Fig. 6).

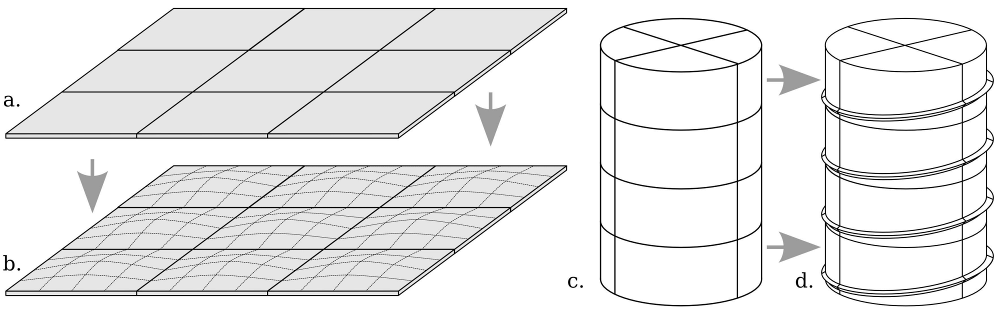

Fig. 5 – Enriched geometry of the master finite element mesh : **a-b** – periodic thin-walled structure, **c-d** – mesh of a screw with 4 turns, represented by enrichment of 16 segments with a screw function $\underline{\rho}_e$ of the master surface $\underline{\rho}$.

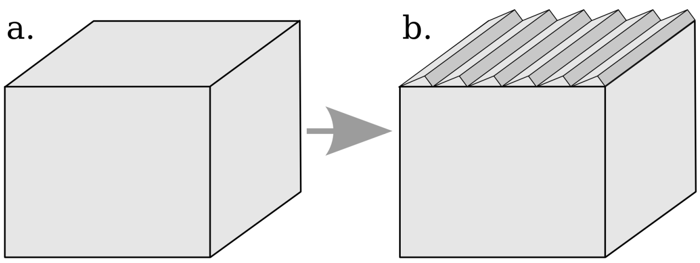

Fig. 6 – **a-b** – An example of contact geometry enrichment for modeling of anisotropic friction.

If one couples the enrichment function with a global smoothing procedure (Fig. 6, c-d), the mentioned shortcoming of the enrichment approach vanishes. Since the master surface is globally smooth, it is not anymore required that the enriching function is zero along the edges of the master segments. However it becomes a real challenge to obtain the needed variations of the geometrical quantities for the resulting enriched surface. On the other hand this coupling makes possible to simulate a shallow wear and to enrich the master geometry independently on the mesh.

The proposed method presents a new general and multipurpose framework in computational contact mechanics and requires further developments and applications.

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